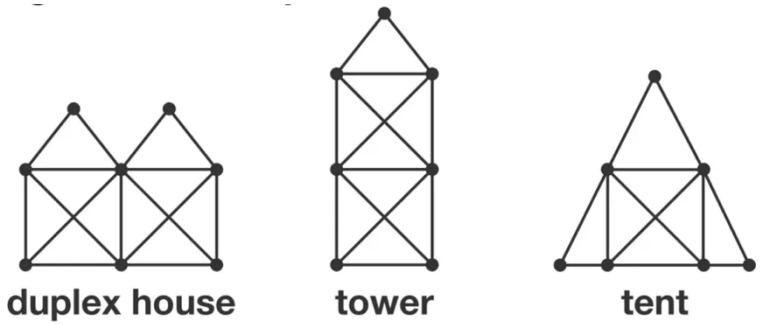
Eulerian Graphs

Which of these graphs below can be drawn with a single stroke of a pen. You cannot draw over the same segment twice.



Answer:

The tent only.

Still left wondering, why?

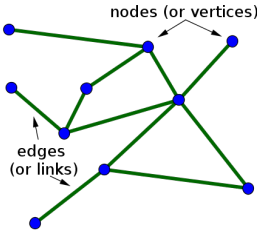
A drawable figure has to be connected and have exactly zero or two nodes of odd degree (can only be a max of 2 nodes with an odd number of edges as these need to be start and finish points).

The Duplex House and Tower both have four odd nodes, while the Tent has only 2 (only the tent has 0 or 2 vertices of odd degree). The tent has 2 vertices with an odd number of paths and the rest are even. Start from one of the odd nodes and it is possible to finish at the other. The others have more than 2 odd nodes.

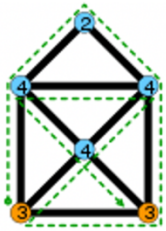
Deeper explanation:

If there is an Eulerian graph then the types of problem described above will work. Eulerian graphs mean you can start at one vertex and travel along each edge exactly once without lifting your pen from the paper.

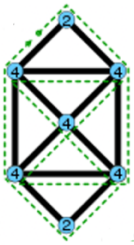
First of all let's define and node and vertex



An Eulerian path is a path that uses every edge of a graph exactly once (allowing for revisiting vertices). An Euler path starts and ends at different vertices. If a graph is connected and has 0 or exactly 2 vertices of odd degree, then it has at least one Euler path



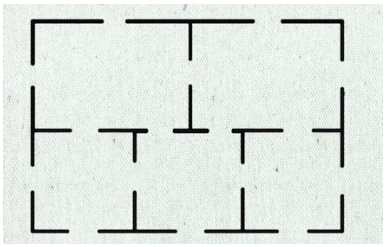
An Eulerian circuit is a circuit that uses every edge of a graph exactly once. An Euler circuit starts and ends at the same vertex (since a circuit). If a graph is connected and has 0 vertices of odd degree, then it has at least one Euler circuit.



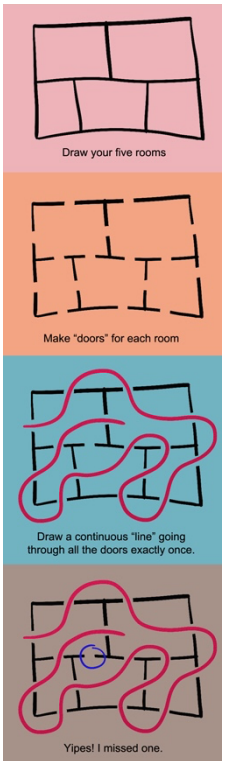
If a graph has more than 2 vertices of odd degree, then it has no Euler paths.

Exercises To Try:

1) Five room house puzzle

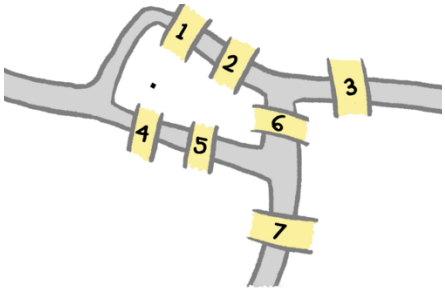


There are 5 rooms. Begin wherever you want and draw one line through all of the doors . You cannot go through same door twice. This is hard, but you should know that it is possible!



2) Bridges of Königsberg

In the German city of Königsberg, there are seven bridges crossing the rivers that meet in the city center. The goal of the puzzle is to find a path that crosses each bridge exactly once (no swimming allowed). You might think to walk a couple of different paths at random or you might even try walking every possible path. Canyou find the path?



1 3 4 5 6 7 2

1 4 5 6 7 2 3

1 5 6 7 2 3 4